Bayesian Knowledge Fusion in Prognostics and Health Management—A Case Study

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Motivation and Background
SAFE-life design assumes very low probability of crack initiation

- Full-scale fatigue tests with 2X safety factors
- **Objective:** Quantify the risk associated with fleet life extension (damage-tolerance regime)
Today’s objective in fleet management is to use an airframe to its maximum service life (total life) [1].

Stochastic Physics-of-Failure (PoF) approach has proved useful for fleet management.

Shortcomings of PoF:
1. Limited knowledge about the underlying physics of failure
2. Scarcity of relevant material-level test data to estimate model parameters
3. In practice, disconnected from the system being modeled (no feedback)

[Rogue flaw] [Assumed initial flaw]

Crack Size

Critical size

Unsafe

Safe

Safe

ΔFH₁

ΔFH₂

ΔFH’

ΔFH₃

Flight Hours

**GOAL:** Developing a hybrid prognostics methodology for health management consisting of the following modules:

- Physics-of-Failure (PoF) Model
- NDI-based structural integrity assessment
- Knowledge Fusion Module
Hybrid PHM Approach

Hybrid Model for fleet management

Crack size \((a)\)

\(a_{\text{crit}}\)

Prediction for \(\DeltaFH\)

Meta model

\[ a = f(FLE \mid \theta) \]

\[ \pi(\theta \mid \text{Evidence}) = \frac{L(\text{Evidence} \mid \theta)p(\theta)}{p(\text{Evidence})} \]
Acoustic Emission Monitoring
Acoustic emissions are elastic stress waves generated by a rapid release of energy from localized sources within a material under stress [3].

- Passive technique (good for detecting damage as it accumulates)
- Global monitoring and localization capability
- Only good for detecting active defects
- Highly susceptible to noise

Deconvolution of the measured voltage signal from the sensor to evaluate the properties of the source event is extremely difficult.

**AE Features**
- Amplitude
- Energy
- Rise time
- Counts (Threshold crossing)
- Frequency content
- Waveform shape
Objective: To correlate AE parameters with fatigue crack growth parameters

\[ \frac{da}{dN} = \beta_1 \log \left( \frac{dc}{dN} \right) + \beta_2 \]

One can estimate \( \frac{da}{dN} \), given \( \beta_1, \beta_2 \) and AE count rate

Experimental Procedure
CT Specimen (7075-T6)

- Crack Growth Clip
Experimental Procedure > Noise Filtration

Noise (AE from any source other that crack growth)
- Rubbing of crack surfaces
- Crack closure
- Grip noise
- Other active defects
Experimental Procedure > Noise Filtration (Hit Type)
Experimental Procedure > Noise Filtration (Peak Freq.)
Experimental Procedure > Noise Filtration

- AE Discovery Tool developed in MATLAB
AE for Fatigue > Model Calibration > Bayesian Regression

Model

\[ Y = \beta_1 X + \beta_2 + \varepsilon \]
\[ \varepsilon \sim \text{Normal}(0, \sigma) \]

Likelihood

\[
L(D|\theta) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{y_i - (\beta_1 x_i + \beta_2)}{\sigma}\right)^2}
\]

Updating parameters via MCMC

Integrating over parameters

\[
f(Y|X,D) = \int \int \int f(Y|X,\theta)\pi(\theta|D)d\theta
\]
\[\theta = \{\beta_1, \beta_2, \sigma\}\]

Probability density of \(da/dN\) For a given value of measured AE count rate and calibrated model parameters.
AE-based Crack Size Prediction

Input AE recording

Logarithmic plot showing input AE recording with cycles (N) on the x-axis and log(ΔN) on the y-axis.

Predicted Growth Rate

Logarithmic plot showing predicted growth rate with cycles (N) on the x-axis and log(ΔN) on the y-axis.

Calibrated Model

Diagram illustrating the process from AE signals to calibrated model.

1. AE Signals
2. Feature Extraction
3. (ΔN)_i
4. (β_1, β_2) / σ
5. Calibrated Model
6. Predicted Growth Rate
7. (Δa)_i
8. Predicted Crack Size
Sources of Uncertainty:
- Stochastic nature of the model
- Initial crack distribution
Knowledge Fusion

Non-destructive Inspection

AE Monitoring of Crack Growth

Feature Extraction
Calibration

Fatigue Crack Growth Parameters ($\Delta K$, $da/dN$)

Crack Size Distribution from AE

Probabilistic AE-Based Diagnostic

Physics of Failure

PoF Model-Based Prediction

Hybrid Prognostic Model

Inspection Field Data

Knowledge Fusion

Bayesian Framework
Acoustic Emission: 
\[ D_{AE} = \left\{ D_{AE}^{(k)} | k = 1 \ldots N_e \right\} \]
\[ D_{AE}^{(k)} = \left\{ (x_k, E^{(k)}) | E^{(k)} \sim f_{E^{(k)}} \left( e^{(k)} | \delta; x_k \right) \right\} \]

Simulation: 
\[ D_{SM} = \left\{ (x_j, y_{ij}) | i = 1 \ldots N_s, j = 1 \ldots N_x \right\} \]
Hierarchical Bayesian Model

\[ \lambda = \{ \theta, \sigma \} \text{ (First level prior)} \]
\[ \phi = \{ M_\theta, \Sigma_\theta \} \text{ (Second level prior)} \]
\[ \Psi = \{ M_{M\theta}, \Sigma_{M\theta}, \Omega, \eta, \alpha, \beta \} \text{ (Vector of hyperparameters)} \]

\[
M_\theta \sim \text{Normal}(M_{M\theta}, \Sigma_{M\theta}) \\
\Sigma_\theta \sim \text{Wishart}(\Omega, \eta) \\
\tau \sim \text{Gamma}(\alpha, \beta) \\
\theta \sim \text{Normal}(M_\theta, \Sigma_\theta) \\
Y \sim \text{Lognormal}(\mu, \tau)
\]

\[ \mu = f(X; \theta) \]
The objective is to infer the model parameters from the simulation and the AE data.

\[
p(\lambda, \phi | D_{SM}, D_{AE}) = ?
\]

\[
p(\lambda, \phi | D_{SM}, D_{AE}) = p(\lambda, \phi | D_{SM}, D_{AE}^{(1)}, \ldots, D_{AE}^{(N_e)}) = p(\lambda, \phi | D_{SM}, E^{(1)}, \ldots, E^{(N_e)})
\]

\[
= \int f_{E^{(1)}}(e^{(1)})p(\lambda, \phi | D_{SM}, e^{(1)}, E^{(2)}, \ldots, E^{(N_e)})de^{(1)}
\]

\[
= \int \int f_{E^{(1)}}(e^{(1)})f_{E^{(2)|E^{(1)}}}(e^{(2)} | e^{(1)})p(\lambda, \phi | D_{SM}, e^{(1)}, e^{(2)}, E^{(3)}, \ldots, E^{(N_e)})de^{(2)}de^{(1)}
\]

\[
= \int \int \cdots \int f_{E^{(1)}}(e^{(1)})\prod_{k=2}^{N_e} f_{E^{(k)|E^{(k-1)}}}(e^{(k)} | e^{(k-1)})p(\lambda, \phi | D_{SM}, e^{(1)}, e^{(2)}, \ldots, e^{(N_e)})de^{(k)}
\]

The correlation between AE data at subsequent time instances is captured in the conditional PDF terms that appear in the above equations.
Now using Bayes’ rule:

\[ p(\lambda, \phi | D_{SM}, e^{(1)}, e^{(2)}, \ldots, e^{(N_e)}) \propto p(D_{SM}, e^{(1)}, e^{(2)}, \ldots, e^{(N_e)} | \lambda, \phi)p(\lambda, \phi) \]

\[ = p(D_{SM} | \lambda, \phi)p(e^{(1)} | \lambda, \phi) \ldots p(e^{(N_e)} | \lambda, \phi)p(\lambda, \phi) \]

Simplification based on the independence of \( D_{SM} \) and \( D_{AE} \) and conditional independence of \( E^{(k)} \)'s given the model parameters, i.e.

\[ p\left(E^{(k_1)}|E^{(k_2)}, \lambda, \phi\right) = p\left(E^{(k_1)}|\lambda, \phi\right), \forall k_1, k_2 = 1, \ldots, N_e \]

\[ = p(D_{SM} | \lambda)p(e^{(1)} | \lambda) \ldots p(e^{(N_e)} | \lambda)p(\lambda | \phi)p(\phi; \psi) \]

\( \phi \) does not appear in the likelihood functions and by using the rules of conditional probability

At this level, each of the likelihood terms, \( p(. | \lambda) \), can be easily calculated as follows:

\[ p(D_{SM} | \lambda) = \prod_{j=1}^{N_x} \prod_{i=1}^{N_s} f_Y(y_{ij} | \lambda; x_j) \]

\[ p(e^{(k)} | \lambda) = f_{Y_k}(e^{(k)} | \lambda; x_k) \]
Knowledge Fusion > Bayesian Model Updating > Solution Approach

- $D_{SM}$ and $D_{AE}$ are independent so we can do sequential updating: First update using $D_{SM}$, use the resulting posterior as the prior for updating with $D_{AE}$

- For $D_{AE}$, discretize the distribution of $E^{(k)}$’s and treat each resulting point as regular evidence. Perform the updating but weigh the resulting posterior using an appropriate weight calculated from the conditional distribution $f_{E^{(k)}|E^{(k-1)}}(e^{(k)} | e^{(k-1)})$

- Bayesian updating at each step is performed via MCMC simulation
  - We use WinBUGS software to find the posterior
  - Weight calculation is performed in MATLAB

- Large computation time for large data sets and lower discretization error.
A case study of using Bayesian fusion technique for integrating information from multiple sources in a structural health management problem was presented.

The simulation data was first used to find the model parameters and then, as crack size estimates from AE became available, the model parameters were updated in light of the new evidence.

The mathematical formulation of the problem as well as the setup of the Bayesian inference solution was given.

The solution includes treatment of ‘uncertain’ evidence and also takes into account the correlation between AE observations.

The resulting equations should be solved numerically. Efforts are still under way to provide an efficient computational solution to this problem.
Thanks you!
Questions? Comments?