Thermodynamic Assessment of Fatigue Crack Initiation

Victor Ontiveros
Mehdi Amiri
Mohammad Modarres
Center for Risk and Reliability
University of Maryland, College Park

David T. Rusk
Team Lead, Airframe Risk & Reliability (AIR 4.3.3)

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Outline

- Objective and Research Thrusts
- Thermodynamics of Fatigue Crack Initiation
  - Energy Approach
    - Experiments, Material, Results
  - Entropy Approach
    - Experiments, Material, Results
- Conclusions and Future Steps
Objectives

- Goal: Prediction of fatigue crack initiation based on Strain Energy Expended and Thermodynamic Entropy Generation
- Thermodynamic assessment of fatigue of specimens with stress concentration (center and edge hole specimens)
- Develop a probabilistic model of the Life Expended of Al 7075-T561
Energy Approach

Total strain energy is the summation of plastic and elastic energy

$$\Delta W = \Delta W_p + \Delta W_{e+}$$

Total strain energy is correlated to life with constants $A$ and $B$

$$\Delta W = A N_f^B$$

$\Delta W_p$: plastic strain energy per cycle

$\Delta W_{e+}$: elastic strain energy per cycle in tension

$\Delta \varepsilon_e$: elastic strain range

$\Delta \varepsilon_p$: plastic strain range
Experimental Overview
Experimental Apparatus and Sample Geometry

Heavy-duty uniaxial fatigue testing machines
Rated 100 kN capacity; 30 Hz frequency

Material:
Aluminum 7075-T6, T651
Strain Energy Model

Cumulative Strain Energy Model

• Transformed to Bayesian Regression Form

\[ \log W_{tot} = \log C - m \log L_e + \epsilon \]

- Error modeled as \[ \epsilon = NOR(0, \sigma) \]
- \( C \) and \( m \) – Parameters
- \( W_{tot} \) – Cumulative Total Strain Energy
- \( L_e \) – Life Expended (0 – 100%) \[ L_e = \frac{N}{N_i}; N < N_i \]
- \( N_i \) is the crack initiation, \( N \) is the cycle number
Strain Energy Approach to Crack Initiation

Total Strain Energy

Sample #

Total Strain Energy (MJ/m³)

- 276 MPa
- 310 MPa
Total Strain Energy
Crack Initiation

Strain Energy Life, Expended

Strain Energy (MJ/m$^3$)

Life Expended

- 276 MPa
- 276 MPa
- 310 MPa
- $\times$ 276 MPa
Strain Energy Results

Note:
Some samples are from different batches with different microstructures
Results of both edge notch and single hole samples are included
Entropy Approach to Crack Initiation

Specimen as a thermodynamic system

Fatigue work

Total Entropy = Entropy Exchange + Entropy Generation

\[ dS = d_e S + d_i S \]

Entropy exchange with the surroundings

Entropy generation due to fatigue

Indication of fatigue degradation; related to plastic deformation
Entropy Generation in Fatigue

- All the deformations cause positive entropy generation rate
  \[ \dot{S}_I = \frac{1}{T} \sigma : \dot{\varepsilon}_p - \frac{1}{T} A_k \dot{V}_k - \frac{1}{T} Y \dot{D} - \frac{1}{T^2} q . \text{grad} T \]

- \( \frac{1}{T} \sigma : \dot{\varepsilon}_p \)  Entropy generation due to plastic deformation

- \( \frac{1}{T} A_k \dot{V}_k \)  Entropy generation due to internal variables
  This term is generally associated with the work hardening effect and is almost 5-10% of the plastic strain energy. This is often neglected.

- \( \frac{1}{T} Y \dot{D} \)  Entropy generation due to damage

- \( \frac{1}{T^2} q . \text{grad} T \)  Entropy generation due to heat conduction in the material

\( \dot{S}_I \) – entropy generation rate; \( \sigma \) – stress tensor; \( \dot{\varepsilon}_p \) – plastic strain rate; \( Y \) – elastic energy release rate
\( T \) – absolute temperature; \( V_k \) – internal variable; \( A_k \) – associated thermodynamic forces; \( D \) – damage variable
\( q \) – heat flux; \( W_p \) – cyclic plastic strain energy;
Assessment of Entropy Generation in Fatigue Experiment

Accumulated entropy generation, $s_i$ up to crack initiation time, $t_i$:

$$s_i = \left( \frac{1}{T} : p \right) \frac{1}{T} A_k \dot{V}_k \frac{1}{T} Y \dot{D} \frac{1}{T^2} q \text{grad}T \right) dt$$

Hysteresis Loop

- $\sigma$: stress
- $q$: heat flux
- $T_0$: Ambient Temperature
- $T_{TC}$: thermocouple temperature
Fatigue Life Estimation based on Entropy Generation

- Thermodynamic Entropy and Life Expended are Correlated:

\[ L_e = f(s_i) \]

- Experimental results show a good correlation between entropy generated and life expended

\[ \frac{S}{S_i} \mu \frac{N}{N_i} \]

S= Cumulative entropy generation at a given cycle
S_i=Cumulative entropy at entropy generation
N= Given Cycle
N_i=Cycle to crack initiation
Entropy Generation at Crack Initiation

Entropy Generation

Entropy Generation (MJ/m$^3$-K)

Sample #

- 276 MPa
- 310 MPa
Entropy Generation
Crack Initiation

Entropy Generation, Life Expended

Entropy Generation (MJ/m^3-K)

Life Expended

- 276 MPa
- 276 MPa
- 310 MPa
- ×276 MPa
Conclusions

- Fatigue life assessment based on strain energy expended and thermodynamic entropy generation can provide a physical explanation without a large number of model parameters.
- Test results and data are generic and applicable to a variety of structural geometries and components.
- The accumulated entropy for crack initiation for Aluminum 7075-T6 was found to range between 0.15 to 0.36 MJ/(M^3K) with an average of 0.26 MJ/(M^3K).
- Further experimental work is required to prove the existence of the entropy limit for crack initiation at variable stress amplitude and variable frequency.
- The entropy accumulation shows a linear correlation with the life expended, providing a reliable tool for fatigue life assessment.


Boroński D., Mroziński S., Metal tests in conditions of controlled strain energy density, Journal of Theoretical and Applied Mechanics, 45,4, 2007 p.773-784

Acoustic Emission Based Model Development for Fatigue Crack Growth Prediction

UMD-NAWCAD Research

Azadeh Keshtgar, UMCP, ENRE PhD Student
Masoud Rabiei, UMCP, ENRE PhD (Graduated)
Mohammad Modarres, UMCP (PI)

David T. Rusk, P.E
Team Lead, Airframe Risk & Reliability (AIR 4.3.3)
GLOBAL CAPABILITY

Current periodic fleet inspection practices:

- Labor-intensive, time consuming and expensive
- Subject to human error
- Inspection itself may cause damage

Inspection intervals selected such that an undetected crack will not grow to critical size before the next inspection

High levels of uncertainty regarding current & future damage state of structure drive recurring manual inspection requirements:

- In-situ NDI (Acoustic Emission, Lamb Waves, etc.) can reduce manual inspection requirements by reducing underlying uncertainty.
Current Acoustic Emission systems can detect the presence of growing cracks, but cannot determine the sizes of such cracks:
- Acoustic Signal must be correlated to crack lengths and growth rates
- Must be discriminated above background noise level
- Must be able to account for variable amplitude loading environment

Measurement of small and large cracks allows an AE system to be used to determine future inspection and repair requirements based on true current damage state

UMD has Demonstrated Correlation of AE signal to crack size and growth rate for constant-amplitude loading
OBJECTIVES

- Model development and validation of crack growth rates correlation with AE signals
- Probabilistic AE-model development of in-situ monitoring of small crack growth
- Investigating the sensitivity of AE signal features to crack initiation
Structural Health Management (SHM)

- **Paradigm shift: offline periodic inspections + online SHM**

- Structural health management (SHM) is the online assessment of structural integrity using appropriate NDI technology

- SHM used for:
  - Direct assessment of the state of structural health in real-time
  - Provide feedback from the structure to improve the prediction of the empirical models

*Diagram showing crack size over flight hours with critical size and safe vs. unsafe states.*
AE monitoring: Theory & Background

AE Features
- Amplitude
- Energy
- Rise time
- **Counts** (Threshold crossing)
- Frequency content
- Waveform shape
Correlation between AE features & Fracture parameters

**AE Features**
- Amplitude
- Energy
- Rise time
- Count rate (dc/dN)
- Frequency
- Waveform shape

**Fracture Parameters**
- \( \frac{dc}{dN} \)
- \( \Delta K \)

\[
\frac{dc}{dN} = A_1 (\Delta K)^{A_2}
\]

\[
\Delta K = A_1^{-1/A_2} \left( \frac{dc}{dN} \right)^{1/A_2}
\]

\[
\log \Delta K = \alpha_1 \log \left( \frac{dc}{dN} \right) + \alpha_2
\]

Statistical model development

\[ D = \{(x_i, y_i) \mid x_i = \log(\frac{dc}{dN})_i, y_i = \log \Delta K_i\}_{i=1}^n \]

\[ y_i \sim N(\mu_i, \sigma_i) \]
\[ \mu_i = \alpha_1 x_i + \alpha_2 \]
\[ \sigma_i = \gamma_1 \exp(\gamma_2 x_i) \]

\[ p(D|\alpha_1, \alpha_2, \gamma_1, \gamma_2) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{1}{2}\frac{(y_i - (\alpha_1 x_i + \alpha_2))}{\gamma_1 \exp(\gamma_2 x_i)}\right) \]

\[ p(\Theta|D) = \frac{p(D|\Theta)p(\Theta)}{p(D)} \]

\[ \Theta = \{\alpha_1, \alpha_2, \gamma_1, \gamma_2\} \]

\[ p(D) = \int p(D|\Theta)p(\Theta)d\Theta \]
AE-based Crack Size Estimation

\[
\log \left( \frac{da}{dN} \right) = \beta_1 \log \left( \frac{dc}{dN} \right) + \beta_2
\]
AE-Based Crack Size Estimation

1. AE Signals
2. Feature Extraction
3. Calibrated Model
4. Crack Size (mm)

\[ (\Delta N)_i \]

\[ \frac{dc}{dN} \]

\[ \frac{da}{dN} \]

\[ (\Delta a) \]

\[ (\beta_1, \beta_2) \]

\[ \sigma \]

\[ \log_{10}(\frac{dc}{dN}) \]

\[ \log_{10}(\frac{da}{dN}) \]

\[ (\Delta N) \]

\[ \log_{10}(\frac{da}{dN}) \]

\[ \log_{10}(\frac{dc}{dN}) \]

\[ \log_{10}(\frac{da}{dN}) \]
AE-Based vs. Empirical Crack Growth Model

AE-Based Crack Growth Model

Empirical Crack Growth Model
The necessary information for developing a structural health diagnostic and prognostic (i.e., SHM) solution is often obtained from various sources.
Dynamic State-Space Model

Recursive Bayesian estimation is a probabilistic approach for estimating an unknown probability density function recursively over time using incoming uncertain observation (noisy measurements) and a mathematical process model that describes the evolution of the state variables over time.

Observations at time step \( k: z_k \in \mathbb{R}^m \)

State variable at time step \( k: x_k \in \mathbb{R}^n \)

Key assumptions:
1. States follow a first order Markov process. \( p(x_k | x_{k-1}, x_{k-2}, \ldots, x_1) = p(x_k | x_{k-1}) \)
2. Observations independent given the states. \( p(z_k | x_k, z_{k-1}, \ldots, z_1) = p(z_k | x_k) \)

We are interested in posterior distribution of state \( x_k \), given the time series of past observations:

\[
p(x_k | z_k, z_{k-1}, \ldots, z_1) = ?
\]
Results: Effect of Frequency of Inspections
Prognosis Results

![Graph showing crack size and loading cycles for Case I and Case II.](image)

- **Case I**
- **Case II**

The graph illustrates the predicted crack growth trajectory and RUL prediction, highlighting the critical crack size.
Crack Initiation and Small Crack Growth

Probabilistic model development for steady state crack growth:

Started development of a probabilistic model for small crack growth:

- Small crack length measurement
  - Close-up camera for large cracks
  - Optical microscopy for small cracks
- Correlation between Small crack growth rates and AE signals
Crack Initiation and Small crack measurement

- Cracks less than 1mm long considered to be small
  ( <0.04 in. or <0.001 m)
- Time-lapse photography
  - 50X magnification microscope for small cracks
- Image processing toolbox for crack size measurement
Optical microscopy for small cracks

- Crack Growth 0.019 in.
- 0.0027 in.
Small crack growth rate versus AE count rate

- Linear correlation observed
- Probabilistic prediction model can be achieved with more data

\[ y = 0.0404x - 15.708 \]
\[ R^2 = 0.1705 \]
Small crack growth rate versus AE energy rate

- Linear correlation observed
- Higher $R^2$ and larger slope than count rate
- Energy showed more sensitivity to crack growth

$y = 0.0683x - 16.756$

$R^2 = 0.4919$
Conclusions and Future Steps

- The AE count rates showed a linear correlation with crack growth rate for large cracks.
- The probabilistic model was successfully applied for large crack estimation.
- For small cracks the AE energy rates showed more sensitivity to crack growth.
- Ongoing AE-based experiments to account for different loading conditions.
- Ongoing experiments to investigate the sensitivity of AE signal features to crack initiation.
- Probabilistic model development for small crack length.
Backup Slides
Thermodynamic Entropy as Defined in Fatigue

• Hysteresis Dissipation:

- Energy dissipated due to internal variables
- Energy dissipated due to elastic damage
- Energy dissipated due to plastic deformation (minus the energy dissipated resulting from internal variables)

\[ E - \delta E \]

\[ \varepsilon \]

\[ \sigma \]

\[ A_k V_k \cdot YD \]
Accomplishments for past year

➢ Experimental
  • Implement use of
    • Strain gauges and thermocouples
    • Force and displacement control tests
  • Multiple stress ratios:
    ▪ R = 0, 0.1, 0.4
  • Finite element modeling
  • Implementing Infra Red technique for T

➢ Model development
  • Developed a Bayesian regression framework in MATLAB
    • Move away from WinBUGS
    • Investigating conversion to C

Preliminary IR camera results showing temperature change at edge notch
Expected accomplishments for current year
Ending Sept. 31, 2012

• Experimental
  • Continued optical and electron microscope inspection
  • IR Camera
  • Finite Element Modeling (Temperature)
• Model development
  • User’s guide
  • Conversion to C
  • Fine tuning / updating with additional experimental results
Two heavy-duty uniaxial fatigue testing machines
Rated 22000 lb (100kN) force capacity; 30 Hz frequency
Air cooled hydraulic power pack
Fracture Mechanics and Probabilistic Physics of Failure Laboratory

Heating Chamber

Corrosive Medium Chamber

Strain gauges and Thermocouples
Scatter Reduction

- Different batches
  - Sample etching
  - Single Batch
- Inclusion vs. flaw
  - SEM
- Temperature
  - IR camera
  - Sample alignment

Samples from different batches showing different microstructures
Information ‘Entropy’

• Statistical Entropy
  • Entropy of a macroscopic system consisting of a large number of microscopic identical particles
    \[
    S = k_B \ln \Omega 
    \]
    \[
    S \equiv -k_B \sum_{i=1}^{m} P_i \ln(P_i) 
    \]
    \(k_B\): Boltzmann’s constant
    \(\Omega\): thermodynamic probability
    \(P\): probability of finding a particle in a microstate \(i\)

• Information Entropy
  \[
  S = -K \sum_{i=1}^{m} P_i \ln(P_i) 
  \]
  \(K\): Constant

• Maximum (information) entropy:
  • “maximally noncommittal with regard to missing information”
  • Lagrangian multipliers